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The emphasis of this continuing research effort was switched to discrete-time systems at the beginning of 1975, when the exponential Fourier densities (EFD's) were discovered on group manifolds. While this effort was pressed on to develop the full strength of the EFD's in the period covered by this report, our research took another turn and was mainly directed at the continuous-time bilinear systems again. The improtance of such systems was repeatedly stated in preceding proposals and can not be overemphasized. We recall that optimal filtering of abelian bilinear systems was resolved.

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20. Abstract continued

The existence of a finite-dimensional, recursive estimator was established for nilpotent Lie-algebraic bilinear systems. However, no attempt was made toward displaying the estimator equations.

A major effort was made to derive explicitly the finite dimensional closed-form, recursive filtering equations when the signal process satisfied a nilpotency condition, thus supplying a complete and constructive solution to this class of problems. In this process, we proved that the filter is bilinear as well and, moreover, that it also possesses alanogous nilpotency property. A number of interesting special cases were identified in which the estimator can be alternatively realized via a linear filter followed by a nonlinear postprocessor, a structure that may prove advantageous from the viewpoint of practical implementation.

The research on the near closure property of the EFD's has been continued throughout this report period. More and more computing time was spent on calculating the maximum informational distance between the convolution of an EFD (1) and an EFD (2) and its best fit by an EFD (2). More and more analysis was required to clarify many subtle issues involved in the numerical venture.

A practical application of the EFD's to the satellite attitude determination using star tracker measurements was carried out. The application represents a new approach to spacecraft attitude determination and, perhaps, the first optimal nonlinear filter ever implemented for a real-world system.

AFOSR-TR- 78-1624

Interim Patent Report

and

Interim Scientific Report

on

Detection, Estimation, and Control

on Group Manifolds

No. 4

TO: Air Force Office of Scientific Research
Air Force Systems Command
USAF

FROM: James T. Lo
Department of Mathematics
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SUBJECT: Grant No. AFOSR-74-2671C
Detection, Estimation, and Control
on Group Manifolds
Fourth Interim Patent and Scientific Reports
covering the period:
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I. General

These interim reports cover work carried out by faculty and staff members of the Department of Mathematics during the period 1 June 1977 through 31 May 1978 under Grant No. AFOSR-74-2671C.

Progress was mainly contained in the technical papers and reports listed in Section II. The first three items listed there are attached herewith and the last one will be forwarded as soon as completed. The progress will be summarized in Section IV.

During the report period the following people contributed to the project: Associate Professor James T. Lo and Dr. Shirish Chikte.

II. Publications

1. Optimal Filters for Nilpotent Associate-Algebraic Bilinear Systems, Proc. 6th Annual Allerton Conference on Communication, Control, and Computing, Monticello, Illinois, October 1978 (with Shirish Chikte).
2. Optimal Filters for Bilinear Systems with Nilpotent Lie Algebras, UMBC Mathematics Research Report No. 78-13, UMBC, Baltimore, Md., also submitted for publication (with Shirish Chikte).
3. Optimal Estimation for the Satellite Attitude using Star Tracker Measurements, UMBC Mathematics Research Report No. 78-14, UMBC, Baltimore, Md., also presented at the 1978 NASA Symposium on Flight Mechanics/Estimation Theory, Goddard Space Flight Center, Greenbelt, Md., October 1978.
4. Convolution of Exponential Fourier Densities and Recursive Filtering on the Circle, to be presented at the 1979 IEEE Conference on Decision and Control, San Diego, California, January 1979 (with M. Nishihama).

III. Patents

No patents have been obtained and no applications for patents have been filed as a result of the progress being reported on.

IV. Summary of Progress

The emphasis of this continuing research effort being supported by AFOSR was switched to discrete-time systems at the beginning of 1975, when the exponential Fourier densities (EFD's) were discovered on group manifolds. While this effort was pressed on to develop the full strength of the EFD's in the period covered by this report, our research took another turn and was mainly directed at the continuous-time bilinear systems again. The importance of such systems was repeatedly stated in preceding proposals and can not be overemphasized. We recall that optimal filtering of abelian bilinear systems was resolved by the author and A.S. Willsky in 1972. The existence of a finite-dimensional recursive estimator was established for nilpotent Lie-algebraic bilinear systems. However no attempt was made towards displaying the estimator equations.

A major effort was made in the report period to derive explicitly the finite dimensional, closed-form, recursive filtering equations when the signal process satisfies a nilpotency condition, thus supplying a complete and constructive solution to this class of problems. In this process, we proved that the filter is bilinear as well and, moreover, that it also possesses analogous nilpotency property. A number of interesting special cases were identified in which the estimator can be alternatively realized via a linear filter followed by a nonlinear postprocessor, a structure that may prove advantageous from the viewpoint of practical implementation.

The research on the near closure property of the EFD's, which started in April 1977, has been continued throughout this report period. More and more computing time was spent on calculating the maximum informational distance between the convolution of an EFD(1) and an EFD(2) and its best fit by an EFD(2). More and more analysis was required to clarify many subtle issues involved in the numerical venture. The work is not yet completed and is hoped to be wrapped up soon.

During this report period, a practical application of the EFD's to the satellite attitude determination using star tracker measurements was carried out under a NASA contract. The application represents a new approach to spacecraft attitude determination and, perhaps, the first optimal nonlinear filter ever implemented for a real-world system.

We will, in the following, summarize the ideas and results of each of the publications.

(1) OPTIMAL FILTERS FOR NILPOTENT ASSOCIATE ALGEBRAIC BILINEAR SYSTEMS

In this paper a least squares filtering problem is considered wherein the signal process of interest is generated by a bilinear dynamical system driven by a Gauss-Markov process while the observation process is generated by adding white, gaussian noise to the above Gauss-Markov process.

Explicit solution to the above nonlinear estimation problem is obtained when the matrix algebra associated with this bilinear equation is nilpotent; i.e., the product of more than a certain fixed finite number of matrices in this algebra vanishes. It is shown that the resulting filter consists of a Kalman-Bucy filter followed by a bilinear (time-varying) system which also possesses the nilpotency property.

Due to the above features the filter is seen to be quite suitable for practical realization. On the other hand the dimensionality of the filter is quite high. By means of an illustrative example it is shown how the dimensionality may be reduced by eliminating certain inherent redundancies.

The motivation for this paper comes from the belief that the class of bilinear signal processes considered here can approximate a much wider class of general nonlinear signal processes.

(2) OPTIMAL FILTERS FOR BILINEAR SYSTEMS WITH NILPOTENT LIE ALGEBRAS

This paper discusses a least squares filtering problem in which the signal process to be estimated is generated by a bilinear dynamical system. The problem is illustrated schematically in figure 1 below.

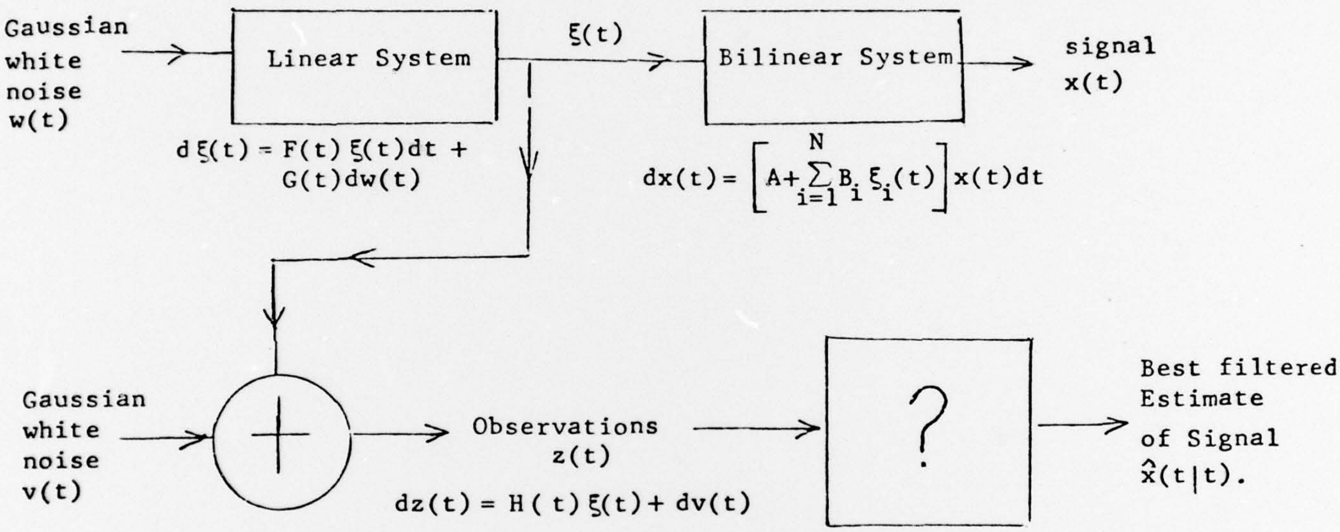

figure 1

A Block Schematic of the Nonlinear Filtering Problem

An exact constructive solution to the above problem is provided under the assumption that the Lie algebra associated with the bilinear system is nilpotent. The resulting filter turns out to be of the form shown in figure 2.

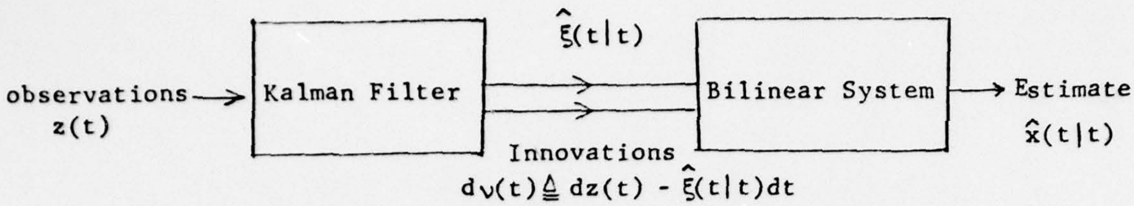

figure 2.

Optimal Filter Structure

Moreover the bilinear system in the filter structure also possesses the nilpotency property.

The dimensionality of the above filter is formidable and some ways of reducing it are demonstrated via an illustrative example. Finally three important special cases are studied in which the bilinear system in figure 2 can be replaced by a simple, memoryless nonlinearity. Possible application of this work include estimation of rotational motion of rigid bodies with one degree of freedom.

(3) OPTIMAL ESTIMATION FOR THE SATELLITE ATTITUDE USING STAR TRACKER MEASUREMENTS

This paper is mainly concerned with estimating the satellite attitude given the gyro readings and the star tracker measurements of a commonly used satellite attitude measuring unit (SAMU). The SAMU is used in such satellites as the high energy astronomy observatory (HEAO) and the precision pointing control system (PPCS). It is composed of 3 to 6 rate gyros and 2 star trackers. The satellite attitude is propagated over a certain number of small time intervals by integrating the satellite angular rates determined from the gyro reading. Gyro drift rates, misalignments, and lack of a precise initial attitude reference then make it necessary to employ two gimbaled star trackers to provide a bench mark to the further propagation of the satellite attitude. A star tracker utilizes an image dissector tube to locate the position of a star on its photosensitive surface. Due to the non-stationary nonlinear characteristics of the image dissector deflection coils and the white noise from the processing electronics, it is at this stage that estimation is required.

A new representation of a probability density of a three dimensional rotation called the exponential Fourier density (EFD), was recently introduced, which has the desirable closure property under the operation of taking conditional distributions. Using the EFD's, an approach was suggested to derive recursive formulas for updating the conditional densities of a rotational process given a nonlinear observation in additive white noise.

In this paper, this approach is carried out for the aforementioned satellite attitude estimation problem. The recursive formulas for updating the conditional densities of the satellite attitude are derived for arbitrary star tracker equations. These general formulas are included here to accommodate possible future consideration of the distortion characteristics of the image dissector deflection coils and possible future change in the star tracker configuration. These general formulas also provide a basis on which special cases can be easily analyzed. However, they involve a large amount of computation. Their feasibility for on-board implementation is highly questionable.

In a conversation with E. J. Lefferts of the GSFC/NASA, it was observed by him that by choosing appropriately the mathematical description of the star tracker configuration, the star tracker measurement can be expressed in closed form as a linear combination of the rotational harmonic functions of order one. This observation substantially simplifies the optimal estimation scheme and greatly reduces the amount of computation required in both designing and utilizing the scheme. A detailed derivation of the associated equations is included in the sequel.

A close look at the mathematical models for the star tracker measurement revealed that the measurement models are not observable. The nonobservability causes a pseudo-image of each observed star. As the extended K-B filtering is merely a local processing, it does not pick up the pseudo-image and is therefore immune from its effect. In contrast, the optimal scheme does not have any "blind spots" and thus assigns an equal probability to the double images of each of the observed stars. An example illustrating such nonobservability is given in Section VI.

Fortunately enough, this difficulty resulting from the nonobservability can be remedied by introducing a "pseudo-measurement" of the second apparent star direction cosine $u_2(k)$ with respect to the tracker base reference axes. We note that $u_2(k)$ is the component of the direction vector $u(k)$ that is perpendicular to the tracker field-of-view and hence can not be measured directly by the tracker. However, from using the satellite attitude estimate $\hat{s}(t)$ at the previous step $t=k-1$, $u_2(k)$ can be predicted, which is to be used as a "measurement" of the real $u_2(k)$. Facilitated with such pseudo-measurement, the pseudo-image of the observed star can be eliminated. For want of a mathematically rigorous proof, only a heuristic explanation for this pseudo-measurement approach, which is believed to be new, is given in Section VII.

The computation required to produce the optimal estimate, involves integrating the conditional covariance matrix of the attitude quaternion, which is very CPU-time-consuming. Encouraging is the fast and high concentration of the conditional probability sensity at the satellite attitude under estimation. This phenomenon is dictated by the theory and suggests two possible ways to get around the difficulty of integration. One way is to localize the integration. Another way is to use the maximum likelihood estimate instead of the optimal estimate. Both methods were researched and implemented on the computer. The simulation results indicate that there is virtually no difference between the estimates obtained in these two ways (at least for the models used in this paper).

The maximum likelihood estimator avoids not only integration altogether but also the task of computing the maximum eigenvalue and its eigenvector. All it needs is the updated Fourier coefficients of the conditional density, which are obtained through simple algebraic formulas. Therefore, the maximum likelihood estimator is used in comparison with the extended K-B filter, a standard method for spacecraft attitude estimation.

A comparison between the K-B filtering and our maximum likelihood estimator was conducted by E. J. Lefferts of the GSFC/NASA. A. N. Mansfield of the CSTA generated a sequence of 33 star tracker observations. The average body rates were provided every one third of a second, and the tracker observation was taken every two minutes. The standard deviation of the tracker measurement noises is 20 arcseconds. For such a low noise level, it is known that linearization is a very good approximation. Therefore, it is not surprising that the maximum likelihood estimator is not much better than the K-B filter.

However, the comparison results indicate that the maximum likelihood estimator is almost always better and converges faster than the K-B filter. It is also noted that: (1) The simulated measurement data are in strict accord with the system model, assuming all the true values of the model parameters and the moise statistics are given. (2) There is no random driving term in the state dynamics model for the spacecraft attitude. Under these two conditions in addition to the low measurement noise level mentioned above, even simple-minded estimators can be expected to perform near optimal. But these conditions are usually far from being met in reality.

The simulated examples in Section IX were chosen to test the robustness of our new schemes and represent tougher working conditions than the real ones. The simulation results to be depicted in graphs indicate that both the local integration estimator and the maximum likelihood estimator track the signal nicely.

The robustness of an estimator toward uncertainty of system parameters and the random driving term in the state dynamics is perhaps the most important consideration in real world application. While there is every reason to believe that the maximum likelihood estimator is superior in this regard, the issue remains to be resolved in the future.

(4) CONVOLUTION OF EXPONENTIAL FOURIER DENSITIES AND FILTERING ON THE CIRCLE

While the EFD(n)'s are closed under conditioning, they are not closed under convolution. This deficiency has prevented us from including random driving terms in the signal processes in our results using EFD(n)'s. In this paper, we will present a striking property of the EFD(n)'s; namely, they are almost closed under convolution.

The maximum informational distance, in the sense of Kullback, between the convolution of two EFD(1)'s and its best fit by an EFD(1) is numerically calculated and is 0.00539412984011850. Such a small number indicates that the convolution of any two EFD(1)'s is virtually an EFD(1). Hence it is not surprising that replacing the convolution of two a priori EFD(1)'s with its best fit EFD(1) yields a near optimal estimate, which is almost indistinguishable from the optimal one. All these numerical results are reported in the paper.

The case of EFD(2)'s will also be thoroughly studied in the paper. More details will be submitted to AFOSR as soon as available.